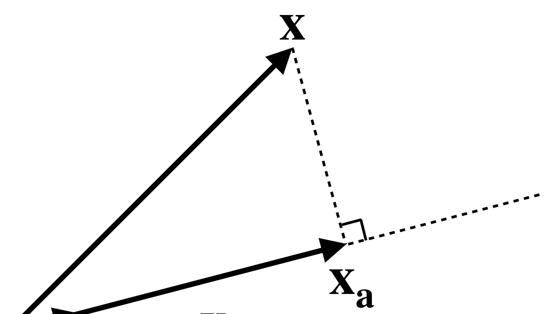
Orthogonal Projector

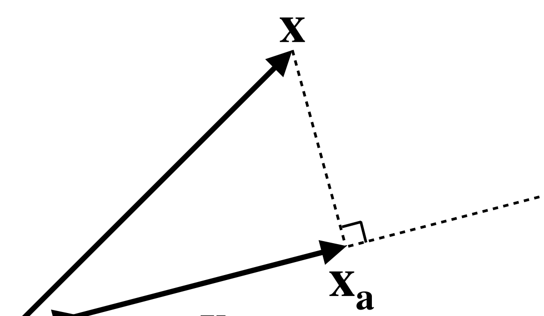
Dexter Studios R&D
Wanho Choi

Orthogonal Projector



$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

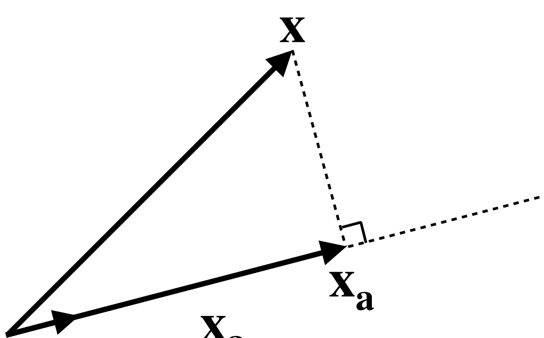
Orthogonal Projector



$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

$$\mathbf{x_a} = (\mathbf{a} \cdot \mathbf{x})\mathbf{a} = (\mathbf{a}^T \mathbf{x})\mathbf{a}$$

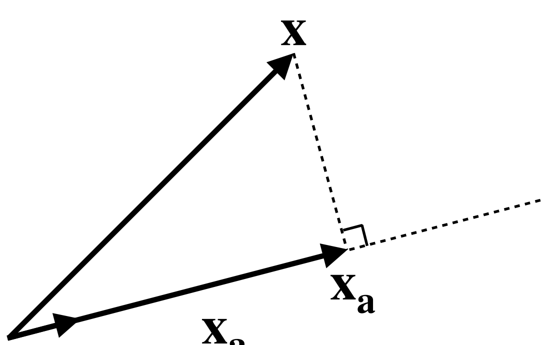
Orthogonal Projector



$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

$$\mathbf{x_a} = (\mathbf{a} \cdot \mathbf{x})\mathbf{a} = (\mathbf{a}^T\mathbf{x})\mathbf{a}$$
$$= \mathbf{a}(\mathbf{a}^T\mathbf{x}) \quad (\because \mathbf{a}^T\mathbf{x} : scalar)$$

Orthogonal Projector



$$\mathbf{a} = \frac{\mathbf{x_a}}{\|\mathbf{x_a}\|}$$

$$\begin{aligned}
\mathbf{x_a} &= (\mathbf{a} \cdot \mathbf{x})\mathbf{a} = (\mathbf{a}^T\mathbf{x})\mathbf{a} \\
&= \mathbf{a}(\mathbf{a}^T\mathbf{x}) \quad (\because \mathbf{a}^T\mathbf{x} : scalar) \\
&= (\mathbf{a}\mathbf{a}^T)\mathbf{x}
\end{aligned}$$

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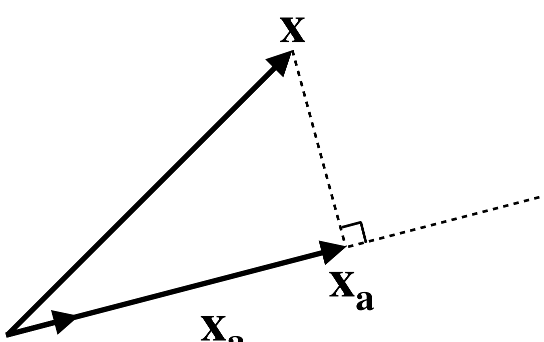

$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

$$\begin{aligned} \mathbf{x_a} &= (\mathbf{a} \cdot \mathbf{x})\mathbf{a} = (\mathbf{a}^T\mathbf{x})\mathbf{a} \\ &= \mathbf{a}(\mathbf{a}^T\mathbf{x}) \quad (\because \mathbf{a}^T\mathbf{x} : scalar) \\ &= (\mathbf{a}\mathbf{a}^T)\mathbf{x} \\ &= \mathbf{P_a}\mathbf{x} \end{aligned}$$

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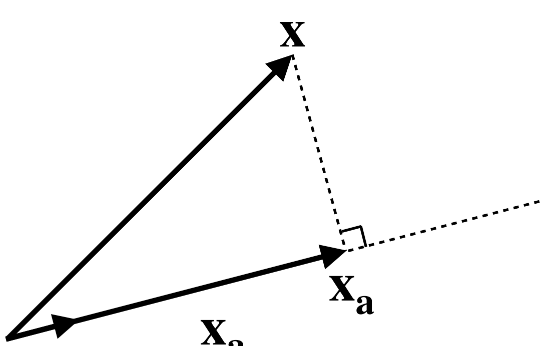

$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

$$\begin{aligned}
\mathbf{x_a} &= (\mathbf{a} \cdot \mathbf{x})\mathbf{a} = (\mathbf{a}^T\mathbf{x})\mathbf{a} \\
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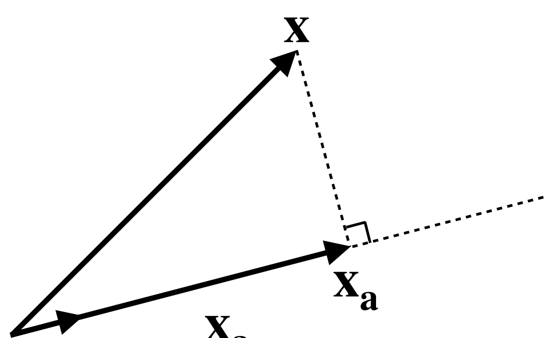
&= (\mathbf{a}\mathbf{a}^T)\mathbf{x} \\
&= \mathbf{P_a}\mathbf{x}
\end{aligned}$$

$$\therefore \mathbf{P_a} = \mathbf{a}\mathbf{a}^T$$

Orthogonal Projector



$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

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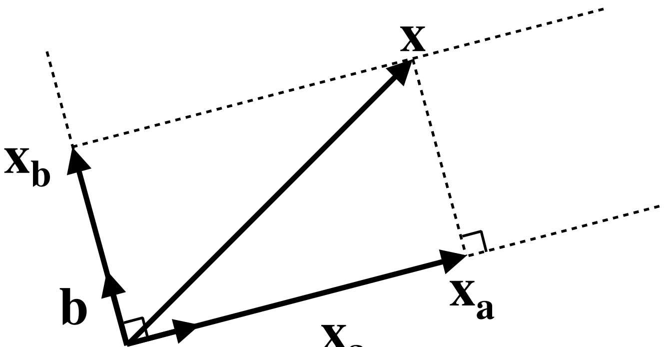
&= \mathbf{a}(\mathbf{a}^T\mathbf{x}) \quad (\because \mathbf{a}^T\mathbf{x} : scalar) \\
&= (\mathbf{a}\mathbf{a}^T)\mathbf{x} \\
&= \mathbf{P_a}\mathbf{x}
\end{aligned}$$

$$\therefore \mathbf{P_a} = \mathbf{a}\mathbf{a}^T \ : projector \ \ (rank = 1)$$

Orthogonal Projector



$$\mathbf{a} = \frac{\mathbf{x_a}}{\|\mathbf{x_a}\|}$$

$$\begin{aligned}\mathbf{x_a} &= (\mathbf{a}\cdot\mathbf{x})\mathbf{a} = (\mathbf{a}^T\mathbf{x})\mathbf{a} \\ &= \mathbf{a}(\mathbf{a}^T\mathbf{x}) \quad (\because \mathbf{a}^T\mathbf{x} : scalar) \\ &= (\mathbf{a}\mathbf{a}^T)\mathbf{x} \\ &= \mathbf{P_a}\mathbf{x}\end{aligned}$$

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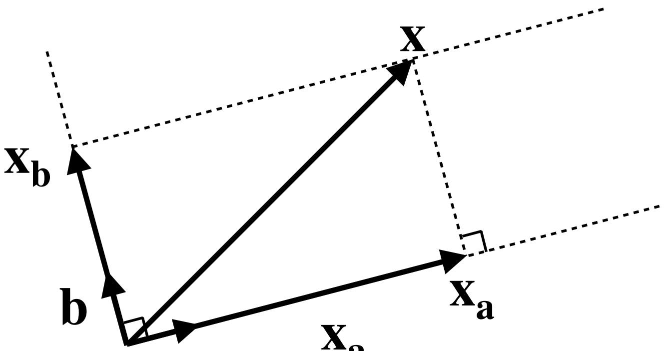
Orthogonal Projector



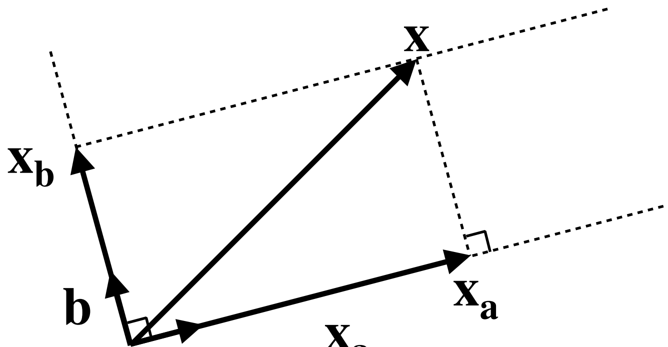
$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

$$\begin{aligned} \mathbf{x_a} &= (\mathbf{a} \cdot \mathbf{x})\mathbf{a} = (\mathbf{a}^T\mathbf{x})\mathbf{a} \\ &= \mathbf{a}(\mathbf{a}^T\mathbf{x}) \quad (\because \mathbf{a}^T\mathbf{x} : \mathit{scalar}) \\ &= (\mathbf{a}\mathbf{a}^T)\mathbf{x} \\ &= \mathbf{P_a}\mathbf{x} \end{aligned}$$

$$\therefore \mathbf{P_a} = \mathbf{a}\mathbf{a}^T \ : \mathit{projector} \ \ (\mathit{rank} = 1)$$

$$\mathbf{x} = \mathbf{x_a} + \mathbf{x_b}$$

Orthogonal Projector



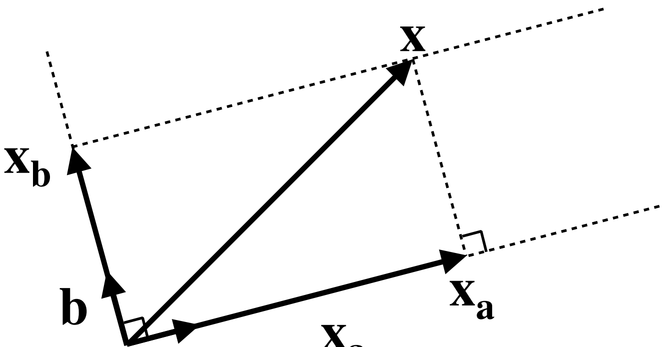
$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

$$\begin{aligned}\mathbf{x_a} &= (\mathbf{a} \cdot \mathbf{x})\mathbf{a} = (\mathbf{a}^T\mathbf{x})\mathbf{a} \\ &= \mathbf{a}(\mathbf{a}^T\mathbf{x}) \quad (\because \mathbf{a}^T\mathbf{x} : scalar) \\ &= (\mathbf{a}\mathbf{a}^T)\mathbf{x} \\ &= \mathbf{P_a}\mathbf{x}\end{aligned}$$

$$\therefore \mathbf{P_a} = \mathbf{a}\mathbf{a}^T \ : projector \ \ (rank = 1)$$

$$\begin{aligned}\mathbf{x} &= \mathbf{x_a} + \mathbf{x_b} \\ &= (\mathbf{a}\mathbf{a}^T)\mathbf{x} + \mathbf{x_b}\end{aligned}$$

Orthogonal Projector



$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

$$\begin{aligned}
\mathbf{x_a} &= (\mathbf{a} \cdot \mathbf{x})\mathbf{a} = (\mathbf{a}^T\mathbf{x})\mathbf{a} \\
&= \mathbf{a}(\mathbf{a}^T\mathbf{x}) \quad (\because \mathbf{a}^T\mathbf{x} : scalar) \\
&= (\mathbf{a}\mathbf{a}^T)\mathbf{x} \\
&= \mathbf{P_a}\mathbf{x}
\end{aligned}$$

$$\therefore \mathbf{P_a} = \mathbf{a}\mathbf{a}^T \;: projector \;\; (rank = 1)$$

$$\begin{aligned}
\mathbf{x} &= \mathbf{x_a} + \mathbf{x_b} \\
&= (\mathbf{a}\mathbf{a}^T)\mathbf{x} + \mathbf{x_b}
\end{aligned}$$

$$\therefore \mathbf{x_b} = \mathbf{x} - (\mathbf{a}\mathbf{a}^T)\mathbf{x}$$

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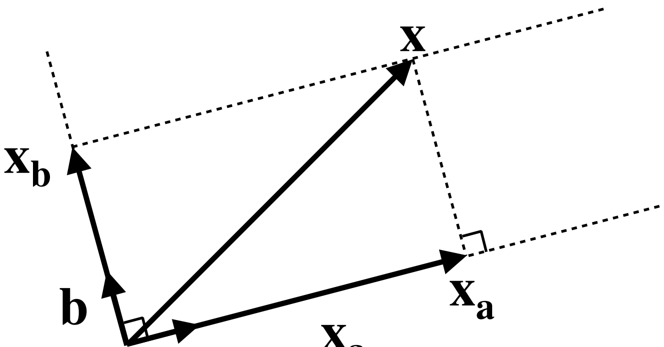


$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

$$\mathbf{x_a} = (\mathbf{a} \cdot \mathbf{x})\mathbf{a} = (\mathbf{a}^T \mathbf{x})\mathbf{a}$$

$$= \mathbf{a}(\mathbf{a}^T \mathbf{x}) \quad (\because \mathbf{a}^T \mathbf{x} : scalar)$$

$$= (\mathbf{a}\mathbf{a}^T)\mathbf{x}$$

$$= \mathbf{P_a}\mathbf{x}$$

$$\therefore \mathbf{P_a} = \mathbf{a}\mathbf{a}^T \ : projector \ \ (rank = 1)$$

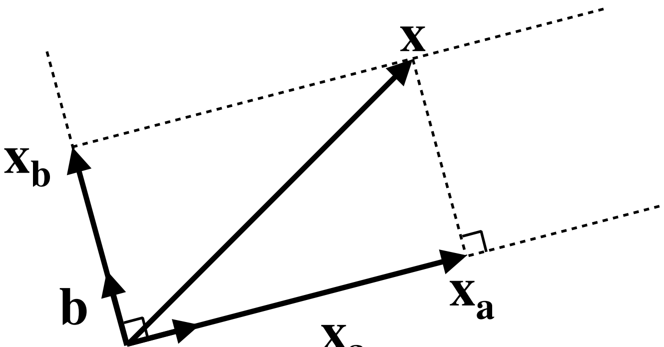
$$\mathbf{x} = \mathbf{x_a} + \mathbf{x_b}$$

$$= (\mathbf{a}\mathbf{a}^T)\mathbf{x} + \mathbf{x_b}$$

$$\therefore \mathbf{x_b} = \mathbf{x} - (\mathbf{a}\mathbf{a}^T)\mathbf{x}$$

$$= (\mathbf{I} - \mathbf{a}\mathbf{a}^T)\mathbf{x}$$

Orthogonal Projector



$$\mathbf{a} = \frac{\mathbf{x_a}}{\| \mathbf{x_a} \|}$$

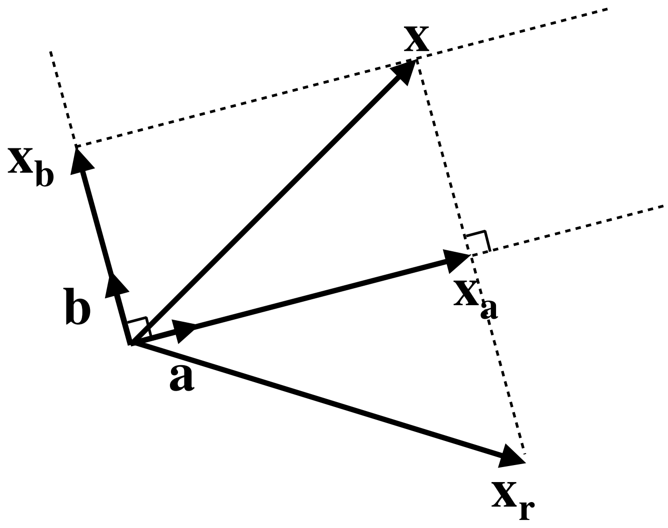
$$\begin{aligned} \mathbf{x_a} &= (\mathbf{a} \cdot \mathbf{x})\mathbf{a} = (\mathbf{a}^T\mathbf{x})\mathbf{a} \\ &= \mathbf{a}(\mathbf{a}^T\mathbf{x}) \quad (\because \mathbf{a}^T\mathbf{x} : scalar) \\ &= (\mathbf{a}\mathbf{a}^T)\mathbf{x} \\ &= \mathbf{P_a}\mathbf{x} \end{aligned}$$

$$\therefore \mathbf{P_a} = \mathbf{a}\mathbf{a}^T \ : projector \ \ (rank = 1)$$

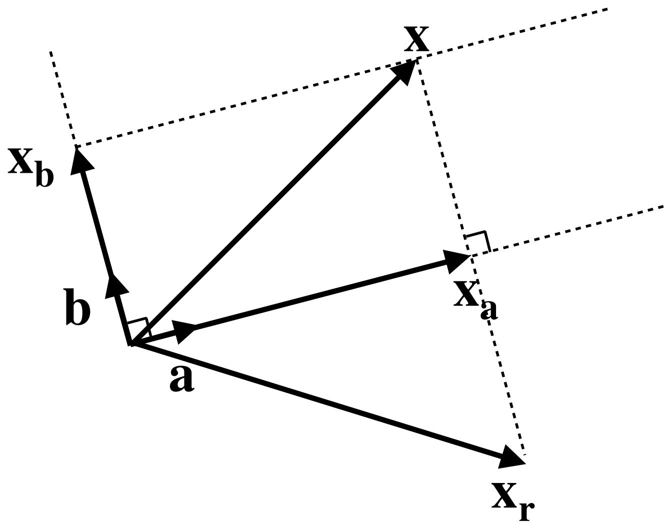
$$\begin{aligned} \mathbf{x} &= \mathbf{x_a} + \mathbf{x_b} \\ &= (\mathbf{a}\mathbf{a}^T)\mathbf{x} + \mathbf{x_b} \end{aligned}$$

$$\begin{aligned} \therefore \mathbf{x_b} &= \mathbf{x} - (\mathbf{a}\mathbf{a}^T)\mathbf{x} \\ &= (\mathbf{I} - \mathbf{a}\mathbf{a}^T)\mathbf{x} \ : complementary\ projector \ \ (rank = 2) \end{aligned}$$

Reflector

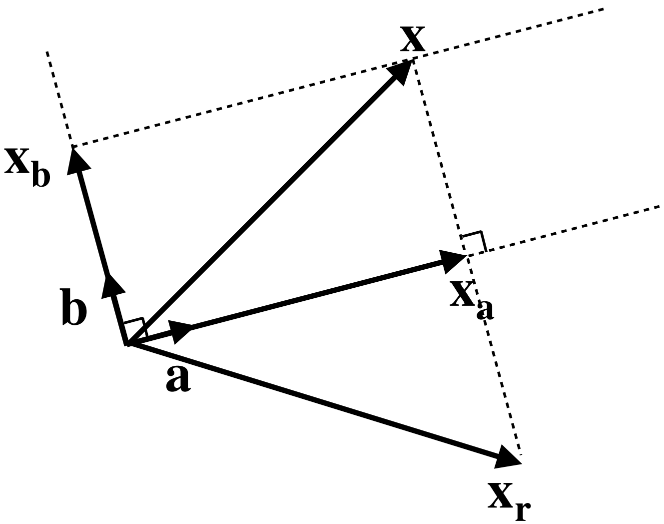


Reflector



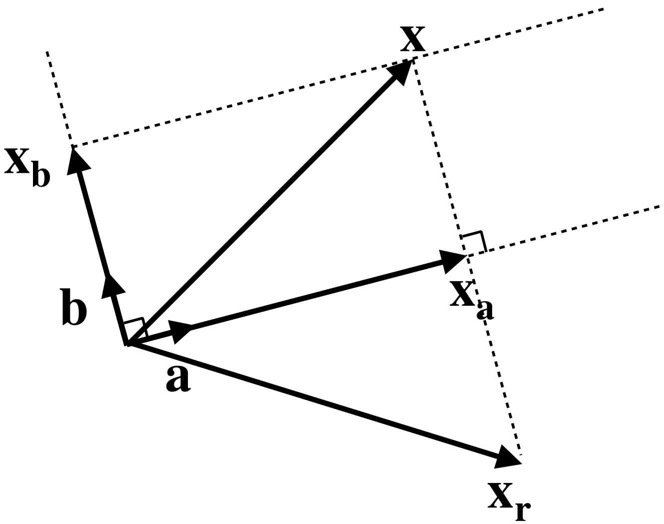
$$\mathbf{x_r} = \mathbf{x} - 2\mathbf{x_b}$$

Reflector



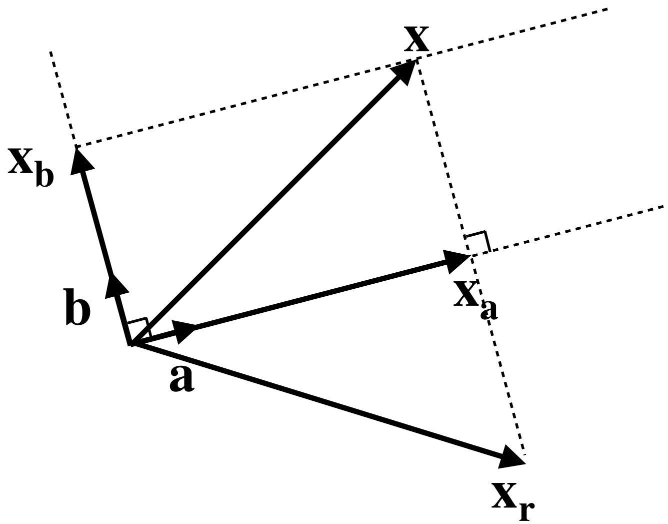
$$\mathbf{x_r} = \mathbf{x} - 2\mathbf{x_b}$$
$$= \mathbf{x} - 2(\mathbf{I} - \mathbf{a}\mathbf{a}^T)\mathbf{x}$$

Reflector



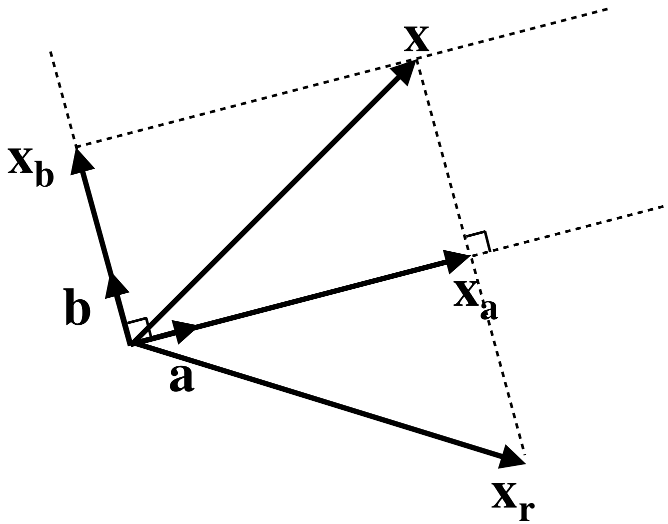
$$
\begin{aligned}
\mathbf{x_r} &= \mathbf{x} - 2\mathbf{x_b} \\
&= \mathbf{x} - 2(\mathbf{I} - \mathbf{a}\mathbf{a}^T)\mathbf{x} \\
&= (\mathbf{I} - 2(\mathbf{I} - \mathbf{a}\mathbf{a}^T))\mathbf{x}
\end{aligned}
$$

Reflector



$$
\begin{aligned}
\mathbf{x_r} &= \mathbf{x} - 2\mathbf{x_b} \\
&= \mathbf{x} - 2(\mathbf{I} - \mathbf{a}\mathbf{a}^T)\mathbf{x} \\
&= (\mathbf{I} - 2(\mathbf{I} - \mathbf{a}\mathbf{a}^T))\mathbf{x} \\
&= (2\mathbf{a}\mathbf{a}^T - \mathbf{I})\mathbf{x}
\end{aligned}
$$

Reflector



$$
\begin{aligned}
\mathbf{x_r} &= \mathbf{x} - 2\mathbf{x_b} \\
&= \mathbf{x} - 2(\mathbf{I} - \mathbf{a}\mathbf{a}^T)\mathbf{x} \\
&= (\mathbf{I} - 2(\mathbf{I} - \mathbf{a}\mathbf{a}^T))\mathbf{x} \\
&= (2\mathbf{a}\mathbf{a}^T - \mathbf{I})\mathbf{x}
\end{aligned}
$$

$$\therefore \mathbf{P_r} = 2\mathbf{a}\mathbf{a}^T - \mathbf{I}$$

Q & A