Matrix

Dexter Studios R&D

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Linear Algebra (선형대수학, 線型代數學)

- A branch of **mathematics** concerning **linear equations** using **vector** and **matrix**

http://hornacek.coa.edu/dave/Teaching/Linear_Algebra.10/

System of Linear Eqns.
(연립 일차 방정식, 聯立一次方程式)

System of equations:

$$\begin{aligned} 2x + 5y &= 10 \\ 3x + 4y &= 24 \end{aligned}$$

Augmented matrix:

$$\left[\begin{array}{cc:c} 2 & 5 & 10 \\ 3 & 4 & 24 \end{array}\right] \begin{array}{l} \leftarrow \text{Eq. 1} \\ \leftarrow \text{Eq. 2} \end{array}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ x & y & \text{constants} \end{array}$$

https://www.khanacademy.org/math/precalculus/precalc-matrices/representing-systems-with-matrices/a/representing-systems-with-matrices

Matrix (행렬, 行列)

- A **rectangular array of numbers** with dimensions **m** (# of rows) by **n** (# of columns).

$$\boldsymbol{A}_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$a_{ij} : \ (i, j) \ element \ (component, \ entry)$$

$$i \in \{1,2,\cdots,m\}$$

$$j \in \{1,2,\cdots,n\}$$

N-Dimensional Vector

- It can be thought as an **n×1 column matrix**:

$$\underset{n \times 1}{\boldsymbol{X}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \underset{1 \times n}{\boldsymbol{X}^T} = \left[x_1, x_2, \cdots, x_n \right]$$

$$\| \boldsymbol{X} \| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

$$A \bullet B = a_1 \times b_1 + a_2 \times b_2 + \cdots + a_n \times b_n$$

Matrix-Matrix Addition

- **Component-wise** addition

$$\boldsymbol{C}_{m\times n} = \boldsymbol{A}_{m\times n} \times \boldsymbol{B}_{m\times n}$$

$$\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

Matrix-Vector Multiplication

- The new vector is the **dot product** of each **row** of the matrix with the **column** vector.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$b_i = \sum_{k=0}^{n} a_{ik} \times x_k$$

Matrix-Matrix Multiplication

$$\boldsymbol{C}_{m\times n} = \boldsymbol{A}_{m\times p} \times \boldsymbol{B}_{p\times n}$$

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pn} \end{bmatrix}$$

$$c_{ij} = \sum_{k=0}^{n} a_{ik} \times b_{kj}$$

Outer Product Matrix

$$\boldsymbol{A}\boldsymbol{B}^T = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} = \begin{bmatrix} a_1b_1 & a_1b_2 & \cdots & a_1b_n \\ a_2b_1 & a_2b_2 & \cdots & a_2b_n \\ \vdots & \vdots & \ddots & \vdots \\ a_nb_1 & a_nb_2 & \cdots & a_nb_n \end{bmatrix}$$

$n \times 1 \quad 1 \times n$ $\qquad\qquad$ $n \times n$

$$A \bullet B = \boldsymbol{A}^T\boldsymbol{B} = a_1 \times b_1 + a_2 \times b_2 + \cdots + a_n \times b_n$$

$1 \times n \quad n \times 1$ $\qquad\qquad$ 1×1

Types of Matrix

Square Matrix	$m = n$	
Diagonal Matrix	$m = n$	$a_{ij} = 0 \quad if \quad i \neq j$
Sparse Matrix	# of zero elements ≫ # of non-zero-elements	
Null Matrix (Zero Matrix)	$a_{ij} = 0$	
Identity Matrix	$m = n$	$a_{ij} = 1 \quad if \quad i = j$ $a_{ij} = 0 \quad if \quad i \neq j$
Transpose Matrix	$\boldsymbol{B} = \boldsymbol{A}^T$	$b_{ij} = a_{ji}$
Symmetric Matrix	$m = n$	$a_{ij} = a_{ji}$
Skew Symmetric Matrix	$m = n$	$a_{ij} = -a_{ji}$
Upper Triangular Matrix	$m = n$	$a_{ij} = 0 \quad if \quad i > j$
Lower Triangular Matrix	$m = n$	$a_{ij} = 0 \quad if \quad i < j$

Q & A