Gram-Schmidt Process

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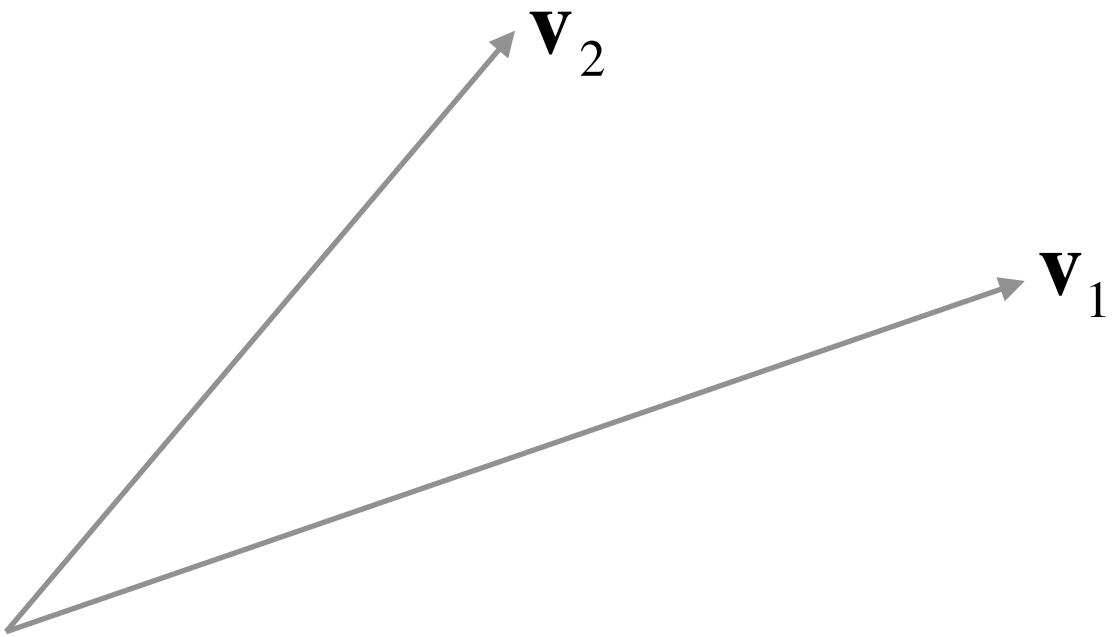
Gram-Schmidt Process

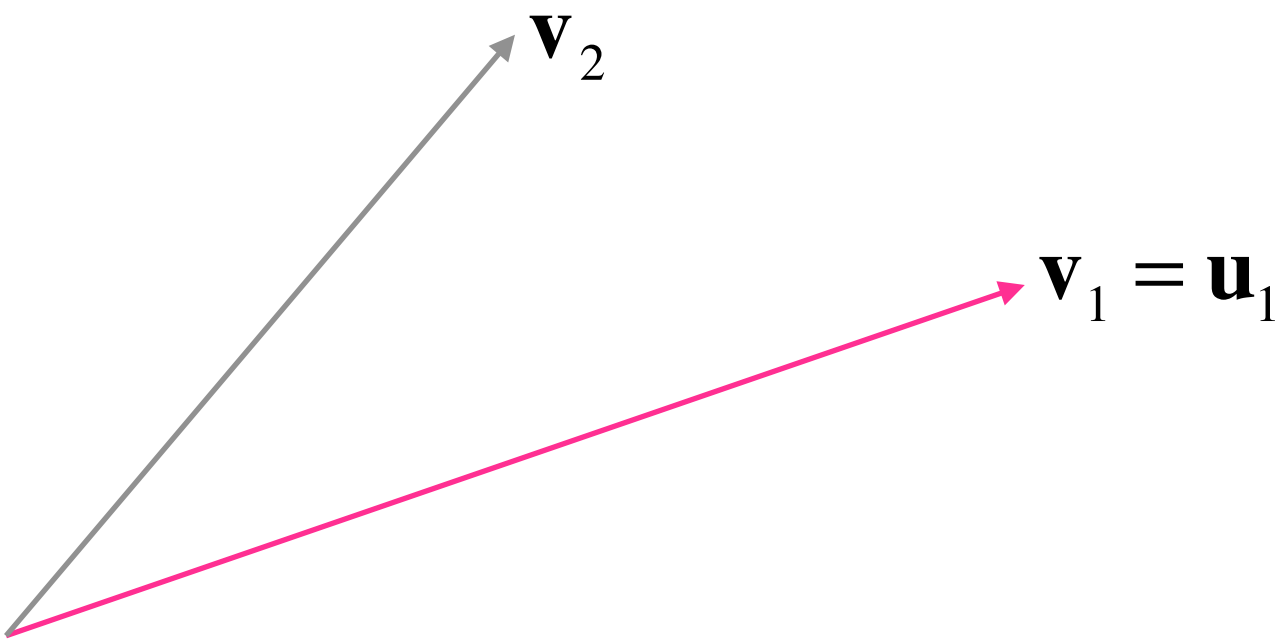
- **Input:** $\{\mathbf{v}_i\}$
  Non-orthogonal set of independent vectors

- **Output:** $\{\mathbf{u}_i\}$
  Orthogonal set of vectors

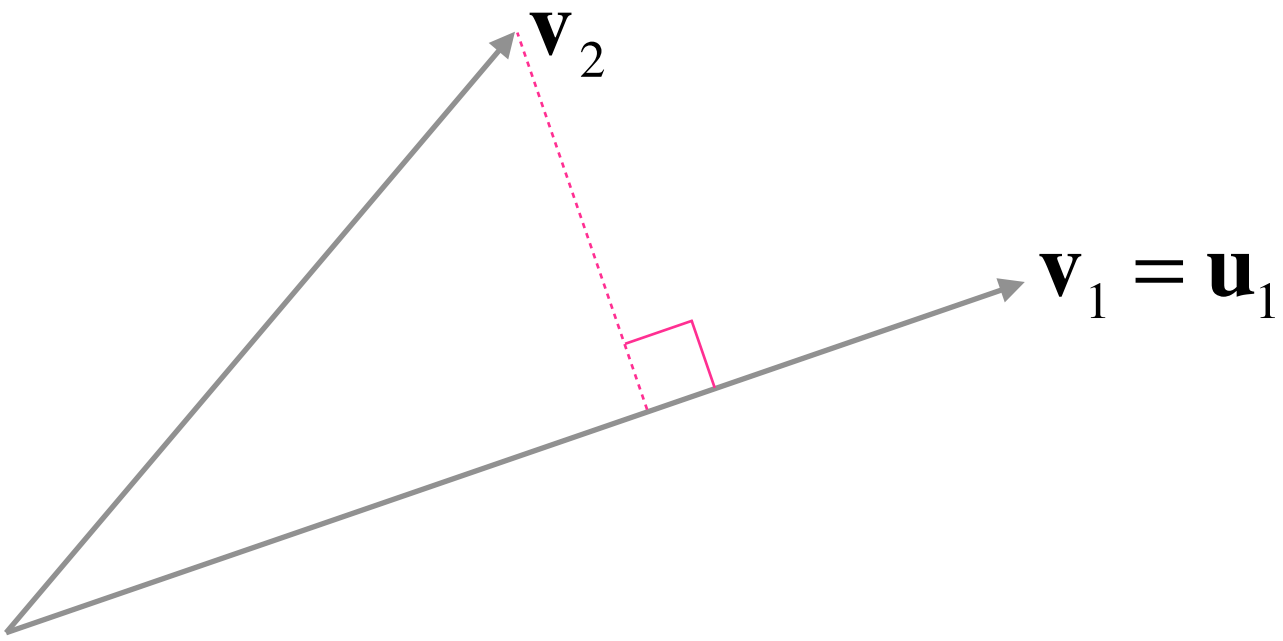
Gram-Schmidt Process



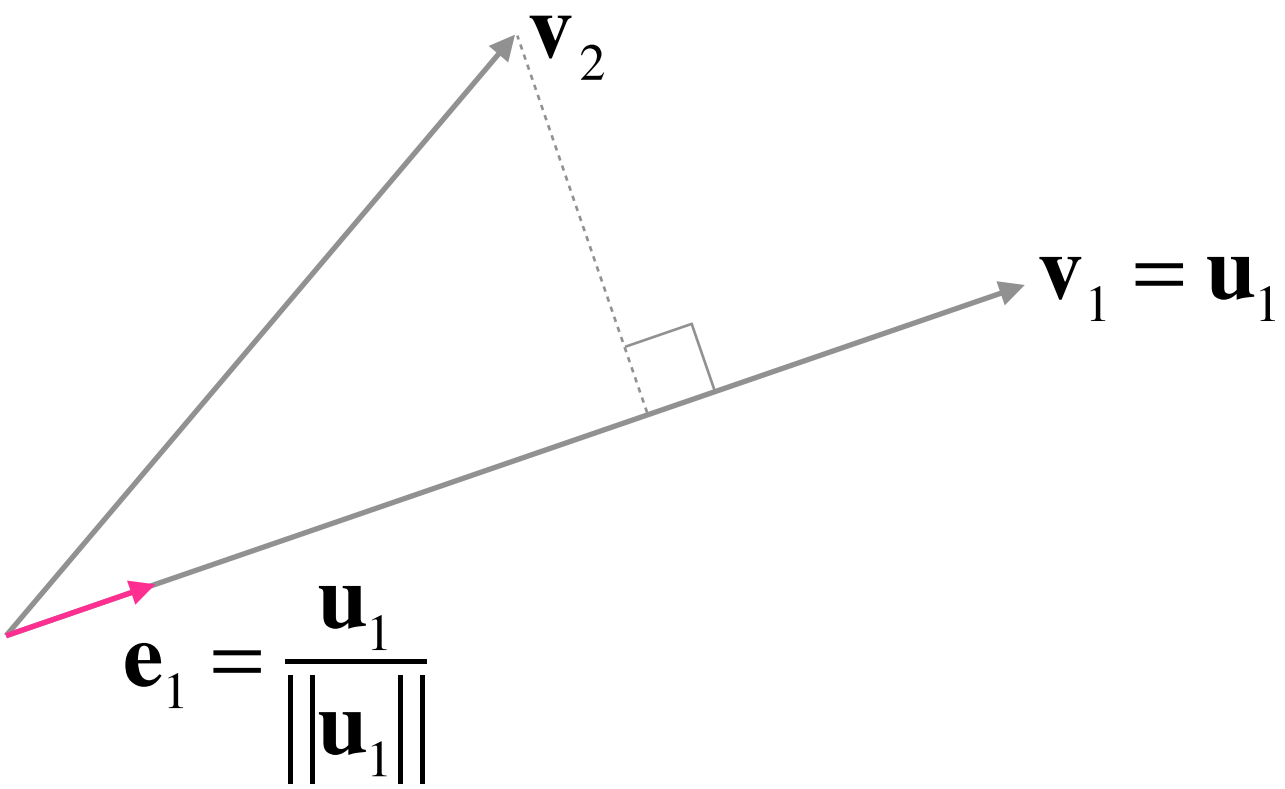
Gram-Schmidt Process



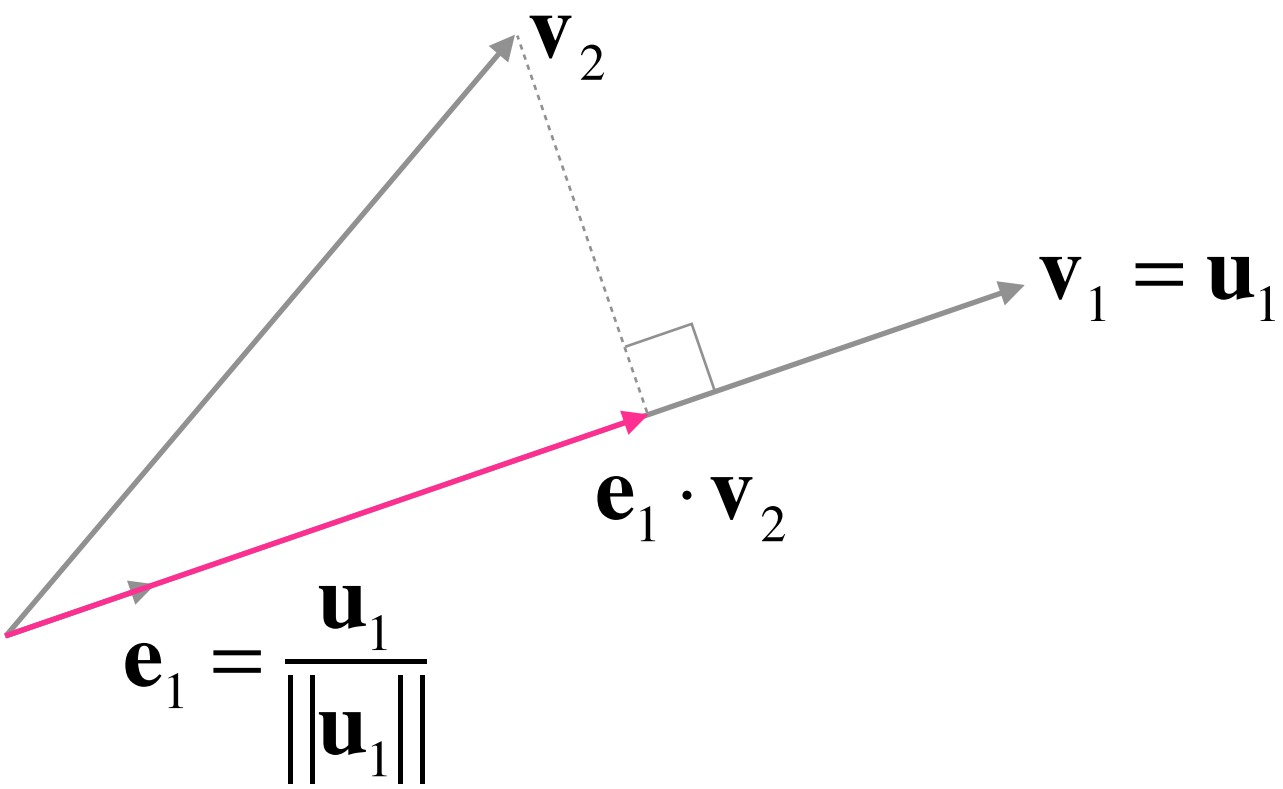
Gram-Schmidt Process



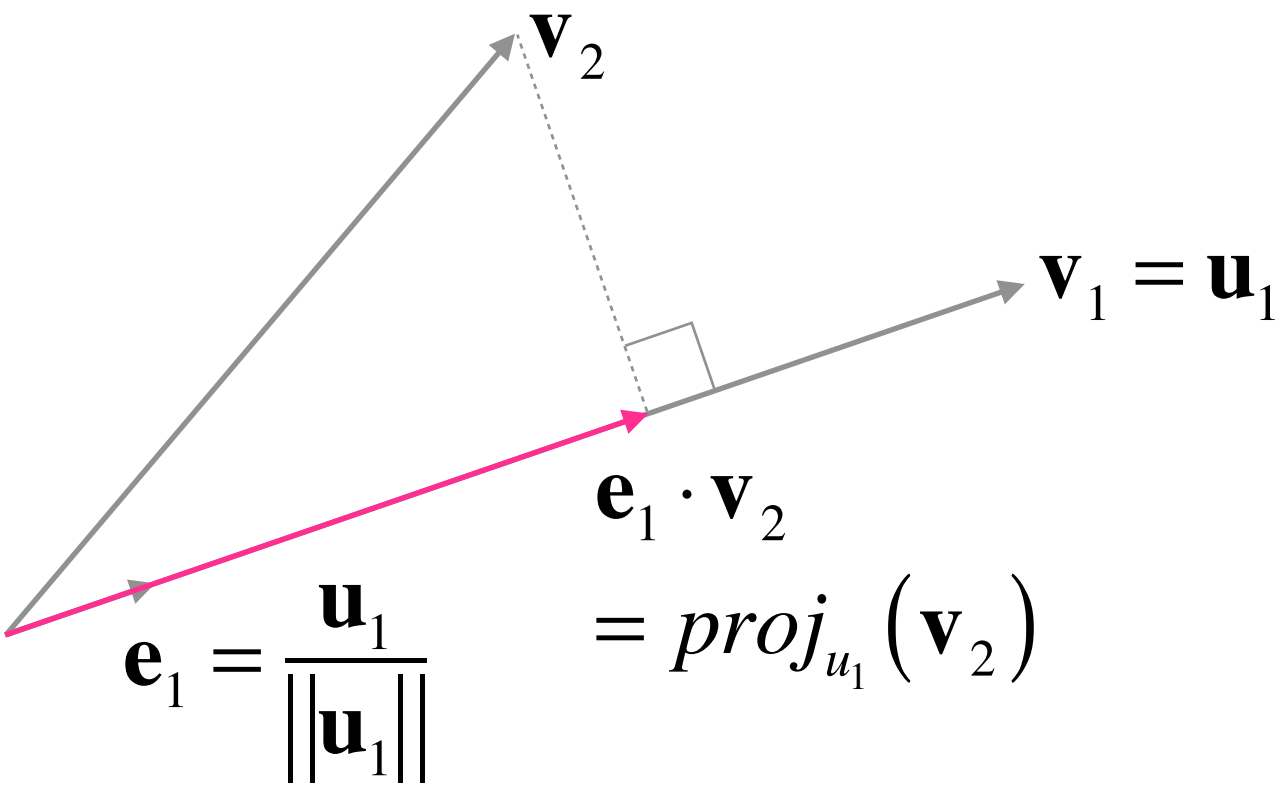
Gram-Schmidt Process



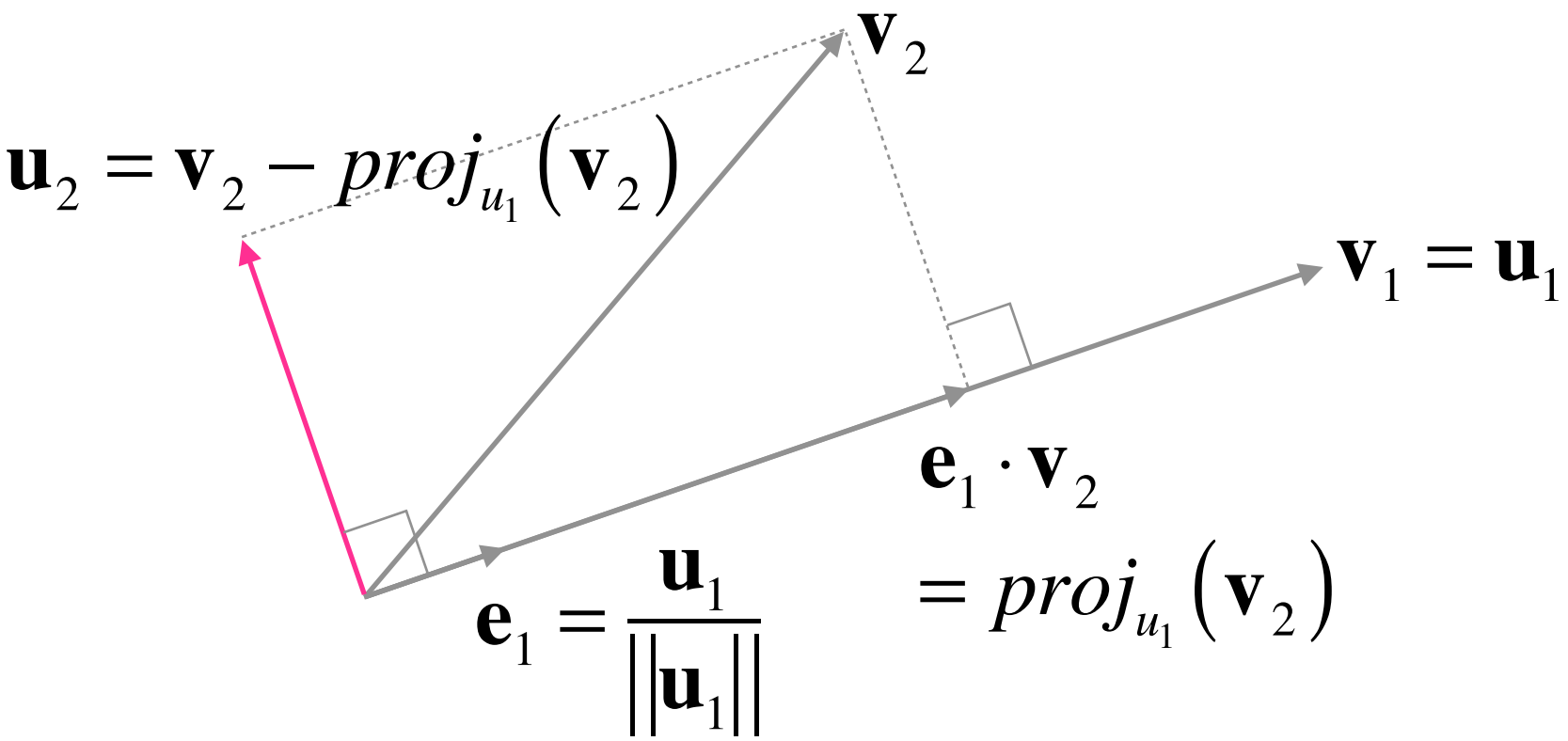
Gram-Schmidt Process



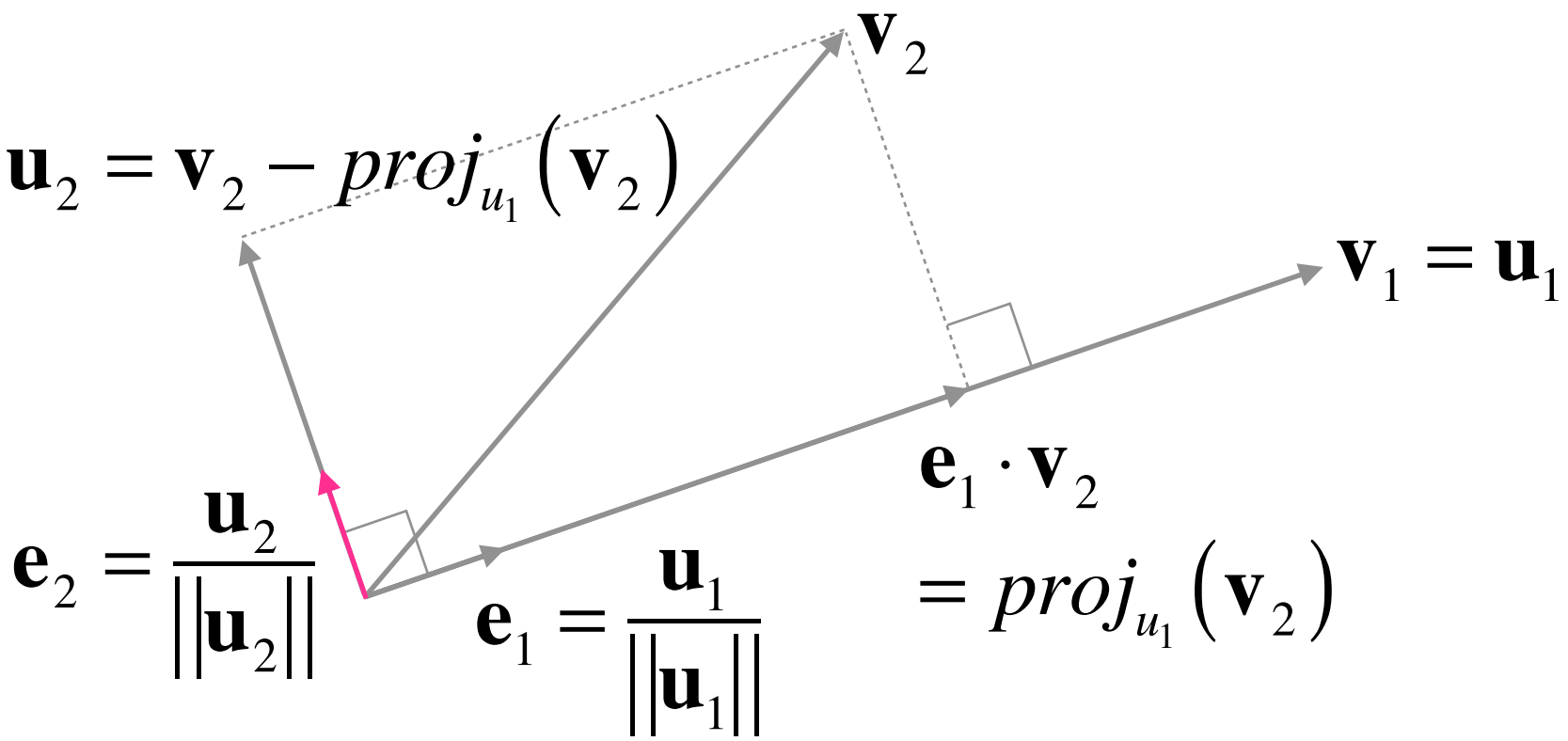
Gram-Schmidt Process



Gram-Schmidt Process



Gram-Schmidt Process



Gram-Schmidt Process

$$\mathbf{u}_1 = \mathbf{v}_1$$

$$\mathbf{u}_2 = \mathbf{v}_2 - proj_{u_1}\left(\mathbf{v}_2\right)$$

$$\mathbf{u}_3 = \mathbf{v}_3 - proj_{u_1}\left(\mathbf{v}_3\right) - proj_{u_2}\left(\mathbf{v}_3\right)$$

$$\vdots$$

$$\mathbf{u}_k = \mathbf{v}_k - \sum_{j=1}^{k} proj_{u_j}\left(\mathbf{v}_k\right)$$

Gram-Schmidt Process

$$\mathbf{u}_1 = \mathbf{v}_1 \qquad \mathbf{e}_1 = \mathbf{u}_1 / \left\| \mathbf{u}_1 \right\|$$

$$\mathbf{u}_2 = \mathbf{v}_2 - proj_{u_1}\left(\mathbf{v}_2\right) \qquad \mathbf{e}_2 = \mathbf{u}_2 / \left\| \mathbf{u}_2 \right\|$$

$$\mathbf{u}_3 = \mathbf{v}_3 - proj_{u_1}\left(\mathbf{v}_3\right) - proj_{u_2}\left(\mathbf{v}_3\right) \qquad \mathbf{e}_3 = \mathbf{u}_3 / \left\| \mathbf{u}_3 \right\|$$

$$\vdots \qquad\qquad \vdots$$

$$\mathbf{u}_k = \mathbf{v}_k - \sum_{j=1}^{k} proj_{u_j}\left(\mathbf{v}_k\right) \qquad \mathbf{e}_k = \mathbf{u}_k / \left\| \mathbf{u}_k \right\|$$

Gram-Schmidt Process

$$\mathbf{u}_1 = \mathbf{v}_1 \qquad \mathbf{e}_1 = \mathbf{u}_1 / \left\|\mathbf{u}_1\right\|$$

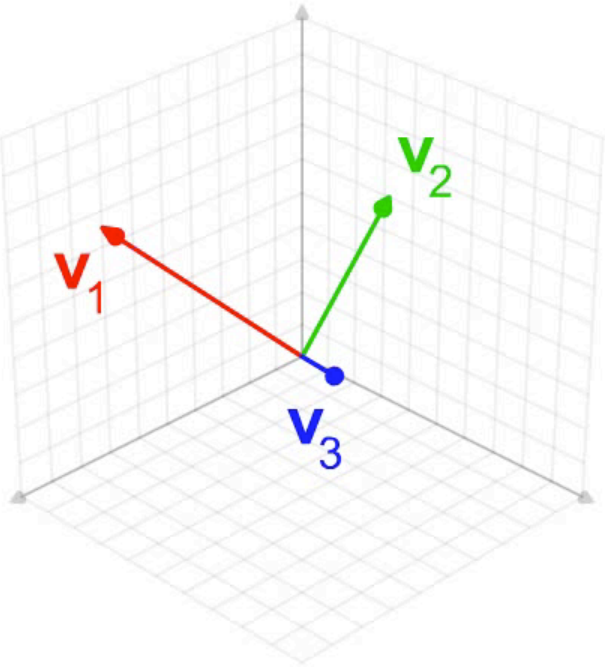
$$\mathbf{u}_2 = \mathbf{v}_2 - proj_{u_1}(\mathbf{v}_2) \qquad \mathbf{e}_2 = \mathbf{u}_2 / \left\|\mathbf{u}_2\right\|$$

$$\mathbf{u}_3 = \mathbf{v}_3 - proj_{u_1}(\mathbf{v}_3) - proj_{u_2}(\mathbf{v}_3) \qquad \mathbf{e}_3 = \mathbf{u}_3 / \left\|\mathbf{u}_3\right\|$$

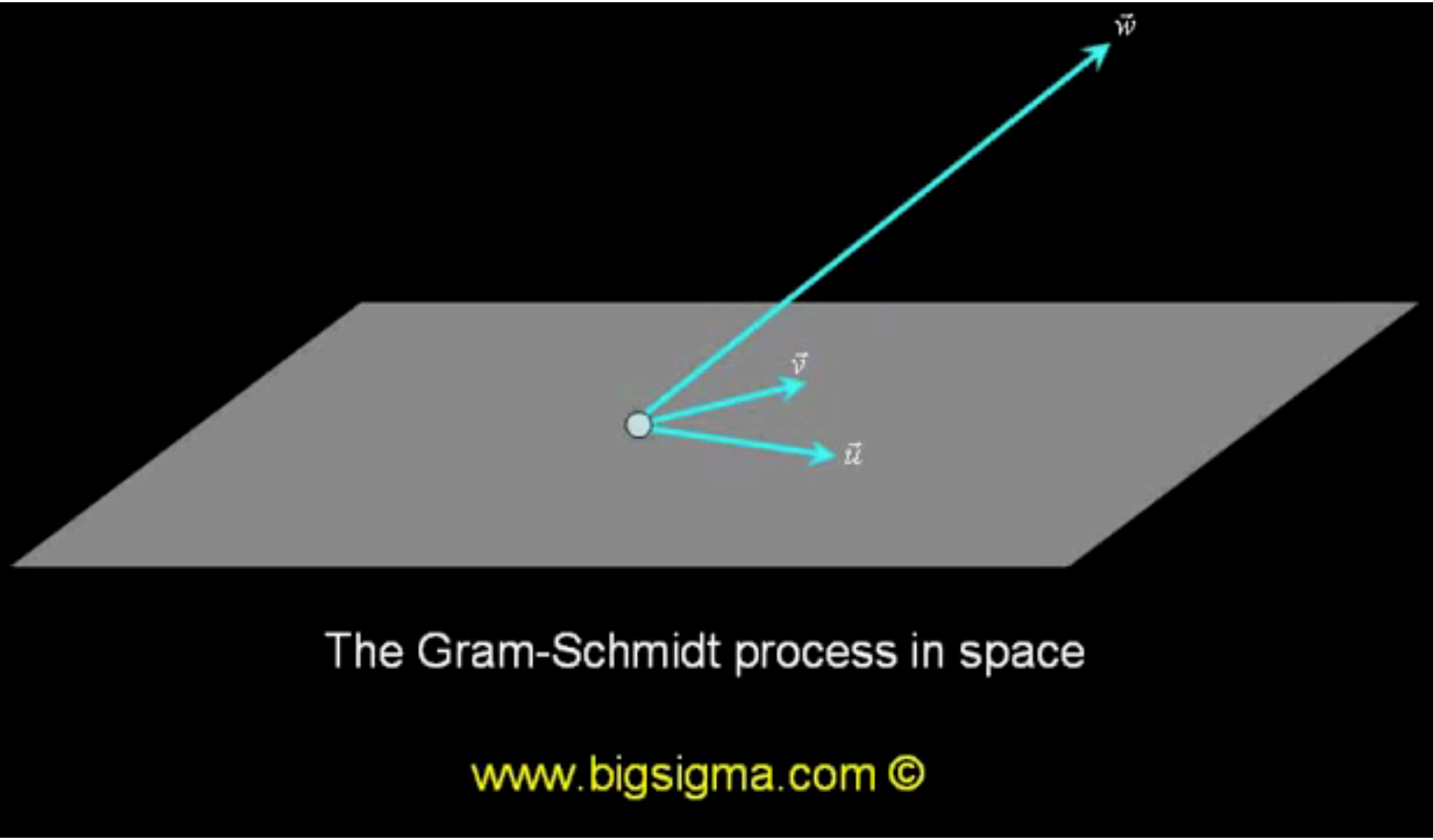
$$\vdots \qquad \vdots$$

$$\mathbf{u}_k = \mathbf{v}_k - \sum_{j=1}^{k} proj_{u_j}(\mathbf{v}_k) \qquad \mathbf{e}_k = \mathbf{u}_k / \left\|\mathbf{u}_k\right\|$$

orthonormal basis



https://www.youtube.com/watch?v=Ys28-Yq21B8



https://www.youtube.com/watch?v=ply8xqh9sWs

Q & A