About Marschner’s Hair Scattering Model

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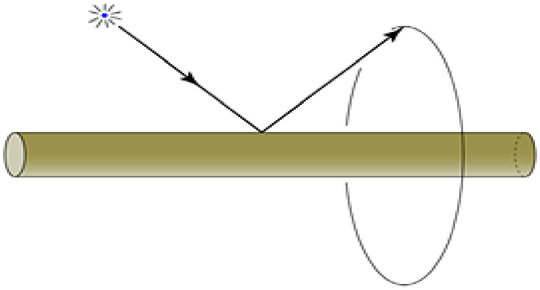
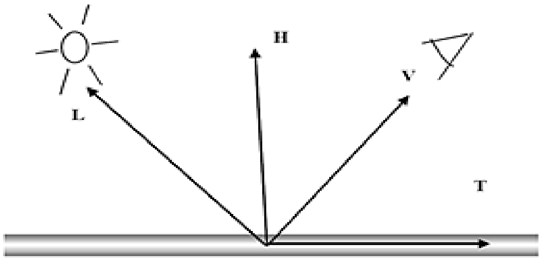
■ Kajiya & Kay’s model (Rendering fur with three dimensional textures, SIGGRAPH 1989)

- A classic phenomenological model

- It use hair strand tangent (T) instead of normal (N).





$$\psi_{diffuse} = K_d \cdot \sin(T, L)$$

$$\psi_{specular} = K_s \cdot sin^{specularity}(T, H)$$

- It does not account for transmission or internal reflection.

- It is not an energy conserving model.

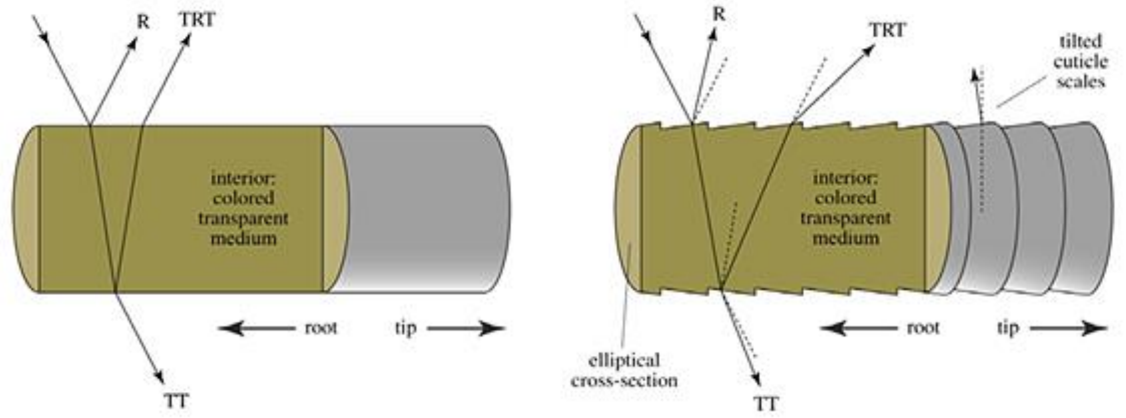
- Real hair is very translucent.

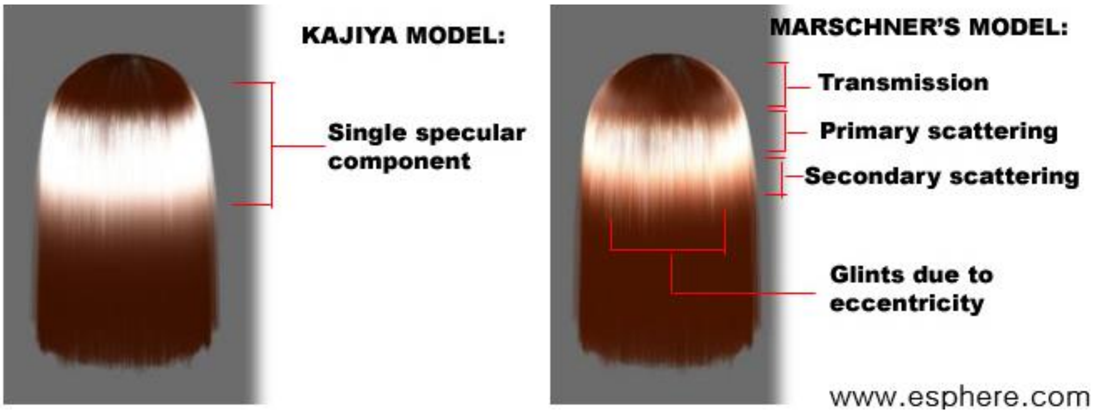
■ Marschner's model (Light Scattering from Human Hair Fibers, SIGGRAPH 2003)

- Out-of-plane scattering (due to the tilted cuticle scales on the hair fiber))

- Multiple specular highlights (due to absorbing interior of the volumetric cylinder)

- Variation in scattering with rotation about the fiber axis (due to elliptical cylinder shape)





■ The storyline of the Marschner's paper

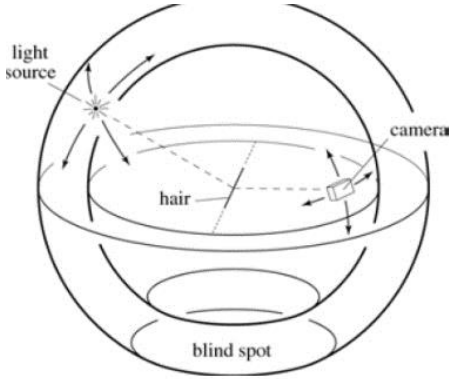
- Experiments & observations

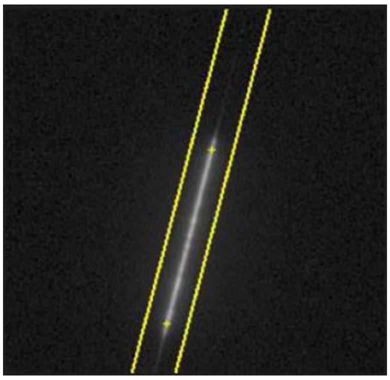
- Approximation based on an analytical scattering function for a circular cylinder

- A practical shading model considering scattering behavior shown in the measurements

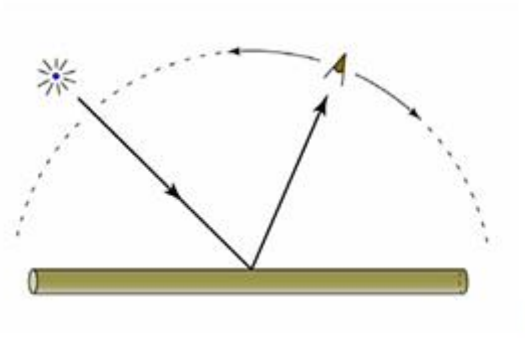
■ Experiments

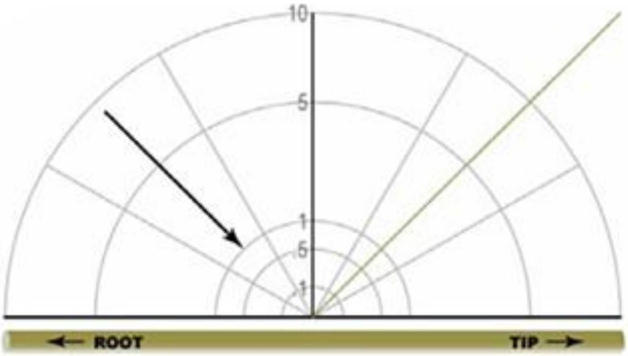
- 4-axes scattering measurements

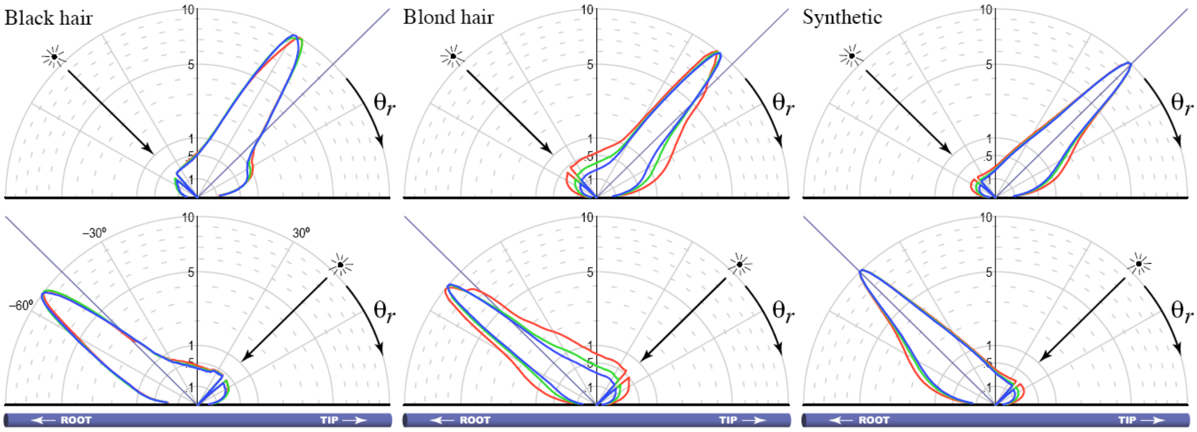




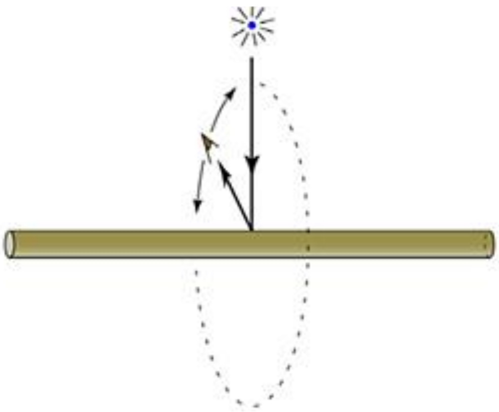
- For the longitudinal directions

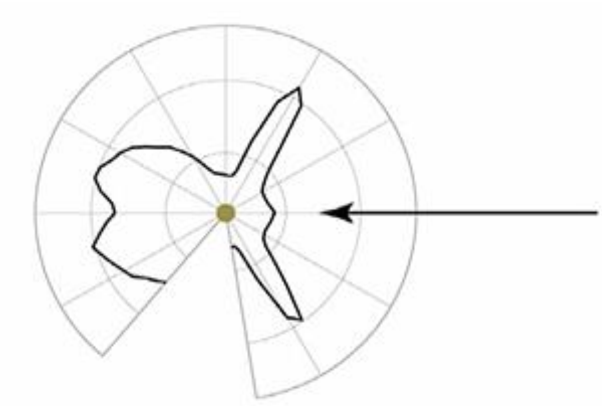


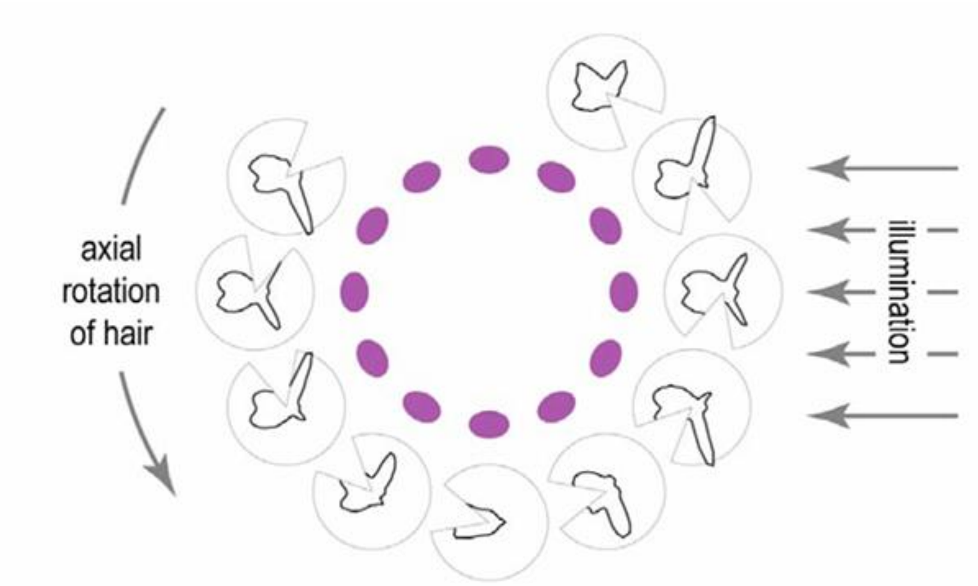




- For the azimuthal directions







■ **Observations**

- Hair is a dielectric material, and very translucent.

- The primary specular highlight continues all the way around the hair.

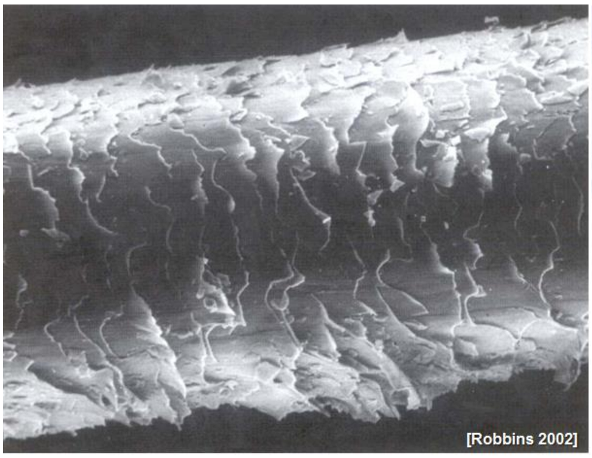
- The secondary highlight is confined to the side of the hair toward the source.

- A pair of large out-of-plane peaks, or glints, is present.

- As the incidence angle increases the peaks move closer to the incidence plane, eventually merging and disappearing.

- Especially the secondary highlight depends on the angle of rotation of the hair fiber about its axis. Therefore, hair fibers are not generally circular in cross section.

■ A real hair fiber



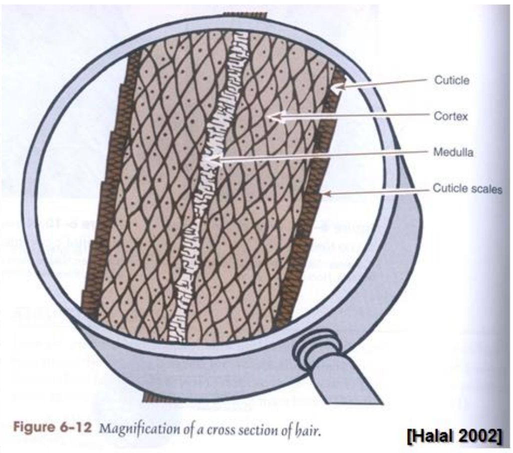
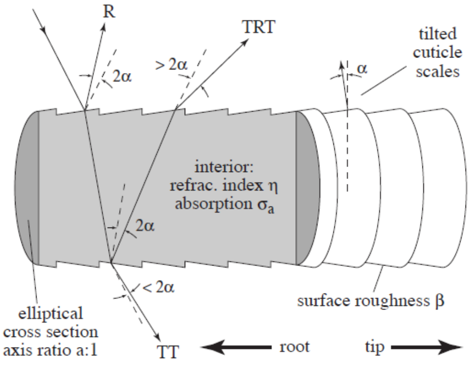
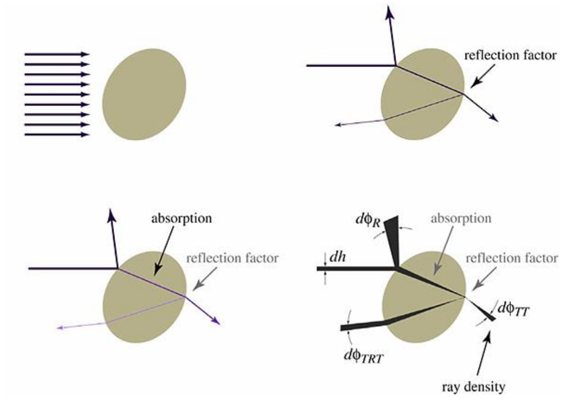


Figure 6-12 *Magnification of a cross section of hair.* **[Halal 2002]**

■ A proposed simple model to explain the observations





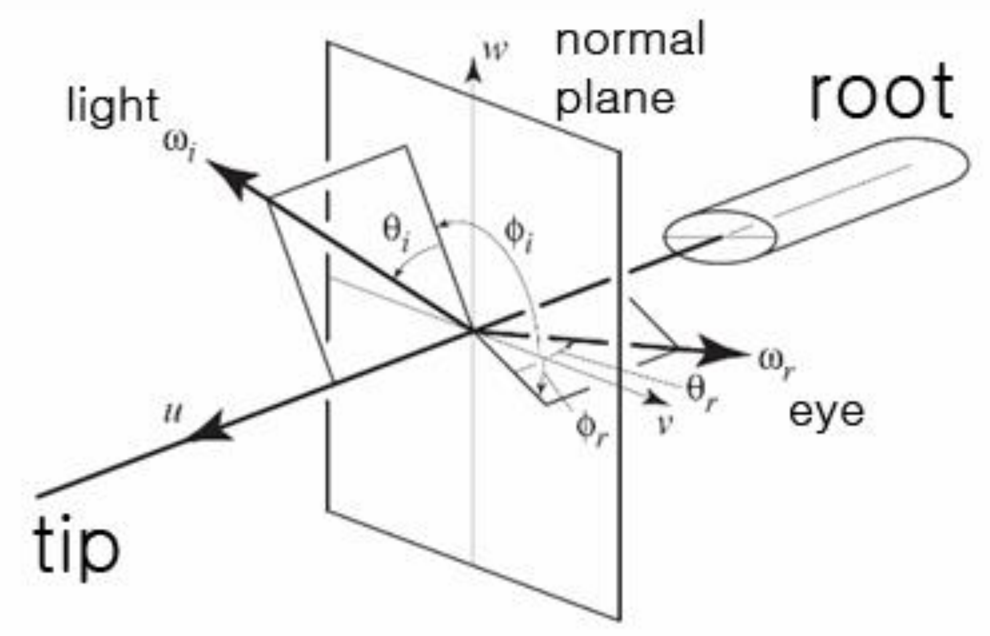
■ Three transport modes

- **R** (surface reflection): A primary highlight, or specular peak toward the root

- **TT** (transmission): A strong forward scattering component that appears in back lighting situation. This makes hair look very bright when lit from behind.

- **TRT** (internal reflection): A colored secondary peak shifted toward the tip

■ Notation for scattering geometry



- $\omega_i(\theta_i, \emptyset_i)$: the direction to the light
- $\omega_r(\theta_r, \emptyset_r)$: the direction to the eye

- θ: the angle in longitudinal direction measured from w-axis (w : 0°, u : +90°).
- ∅: the angle in azimuthal direction measured from v-axis (v : 0°, w : +90°).

※ All variables must have radian unit while calculation.

■ Several derived angles

- $\theta_h = (\theta_r + \theta_i)/2$	: The longitudinal half angle
- $\emptyset_h = (\emptyset_r + \emptyset_i)/2$	: The azimuth half angle
- $\theta_d = (\theta_r - \theta_i)/2$	: The longitudinal difference angle
- $\emptyset = \lvert\emptyset_r - \emptyset_i\rvert$	: The azimuth relative angle

■ Bi-directional Curve Scattering Distribution Function (BCSDF)

$$S = S_R + S_{TT} + S_{TRT}$$

$$S_p = M_p \cdot N_p \quad \text{where} \;\; R(p=0), TT(p=1), TRT(p=2)$$

- M_p: The longitudinal scattering function (θ dependence)

- N_p: The azimuthal scattering function (∅ dependence)

$$\begin{aligned} &S(\theta_i, \emptyset_i; \theta_r, \emptyset_r) \\ &= S_R(\theta_i, \emptyset_i; \theta_r, \emptyset_r) + S_{TT}(\theta_i, \emptyset_i; \theta_r, \emptyset_r) + S_{TRT}(\theta_i, \emptyset_i; \theta_r, \emptyset_r) \end{aligned}$$

- $S_R(\theta_i, \emptyset_i; \theta_r, \emptyset_r) = M_R(\theta_h) \cdot N_R(\eta'(\eta, \theta_d); \emptyset)/cos^2(\theta_d)$

- $S_{TT}(\theta_i, \emptyset_i; \theta_r, \emptyset_r) = M_{TT}(\theta_h) \cdot N_{TT}(\eta'(\eta, \theta_d); \emptyset)/cos^2(\theta_d)$

- $S_{TRT}(\theta_i, \emptyset_i; \theta_r, \emptyset_r) = M_{TRT}(\theta_h) \cdot N_{TRT}(\eta'(\eta^*(\emptyset_h), \theta_d); \emptyset)/cos^2(\theta_d)$

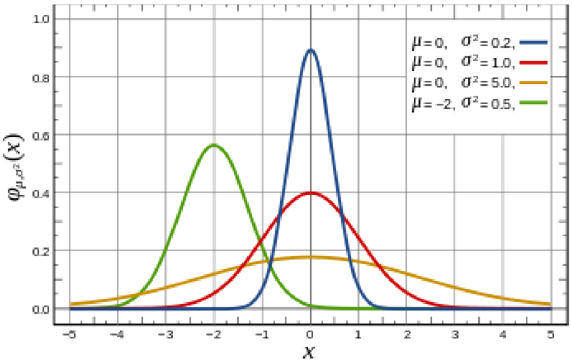
- $cos^2(\theta_d)$ accounts for the projected solid angle of the specular cone.

■ The longitudinal scattering function M

- $M_p(\theta_h)$ can be approximated delta function as a Gaussian.

- The normalized Gaussian function with standard deviation σ is:

$$g(\sigma; x\_\mu) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x\_\mu}{\sigma}\right)^2}$$



- $M_R(\theta_h) = g(\beta_R; \theta_h - \alpha_R)$

- $M_{TT}(\theta_h) = g(\beta_{TT}; \theta_h - \alpha_{TT})$

- $M_{TRT}(\theta_h) = g(\beta_{TRT}; \theta_h - \alpha_{TRT})$

■ The azimuthal scattering function N

- Since we know that all rays originating from a particular incident direction maintain the same inclination to the normal plane, a 2D analysis that works in the normal plane for arbitrary refractive index suffices to describe the 3D scattering function.

- Some tricks are to be used for calculating N.

- N can be analyzed by examining only the projection into a normal plane.

- The projected vectors also obey Snell's law, but two virtual indices of refraction η' and η'' have to be used instead of the index of refraction η (the index of refraction of hair fiber, =1.55).

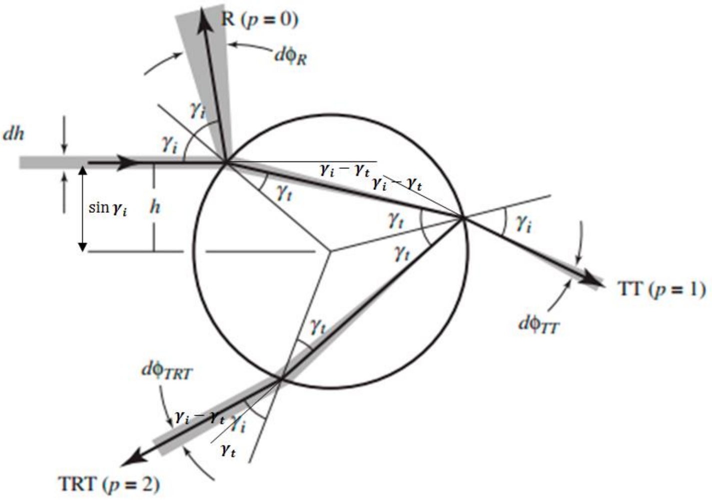
- $\eta'(\eta, \theta_d) = \sqrt{\eta^2 - sin^2(\theta_d)} / \cos(\theta_d)$ [perpendicular component]

- $\eta''(\eta, \theta_d) = \eta^2 / \eta'(\eta, \theta_d)$ [parallel component]

- N_R is the direct reflection.

- N_{TT} and N_{TRT} can be calculated by finding all the paths that contribute to them in a given direction ∅.

- When calculating N_{TT} and N_{TRT} , the attenuation by absorption while traversing inside a hair fiber should be considered



- The exit angle of p-path: $\phi(p, h) = 2p\gamma_t - 2\gamma_i + p\pi$

- To find out all the paths that contribute to scattering in a given direction ∅, we have to solve the equation: $\phi(p, h) - \phi = 0$.

- In the p=0 and p=1 cases, there is a single root, and thus a single path.

- However, for the p=2 case, there may be one or three roots and hence on or three paths.

- Because the formula, $\phi(\mathrm{p},\mathrm{h}) - \phi = 0$, involves Snell's law and hence arcsines, it is computationally expensive to solve for h exactly.

- Therefore, we approximate Snell's law with the cubic polynomial that matches the value and derivative of the exact expression at +90° and -90°: $\gamma_t = (3\mathrm{c}/\pi)\gamma_i - (4c/\pi^3)\gamma_i{}^3$.

- With this approximation, ϕ is a cubic in γ_i: $\widehat{\phi}(p,\gamma_i) = \left(\frac{6pc}{\pi}\right)\gamma_i - \left(\frac{8pc}{\pi^3}\right)\gamma_i{}^3 + p\pi$.

($\phi(\mathrm{p},\mathrm{h}) = 2\mathrm{p}\gamma_t - 2\gamma_i + p\pi \leftarrow \gamma_t = (3\mathrm{c}/\pi)\gamma_i - (4c/\pi^3)\gamma_i{}^3$)

- We also need to estimate the length of each internal path for calculating N_{TT} and N_{TRT}.

- The internal path is $2\cos(2\gamma_i)$ times the radius of the cylinder.

(In the paper, $2 + 2\cos(2\gamma_i)$ is mistakenly used.)

- Therefore, the absorption factor $\mathrm{T}(\sigma_a, \mathrm{h}) = \exp(-\sigma_a\, 2\cos(2\gamma_i))$

- The attenuation factor in front of the intensity contributed by a path:

$\mathrm{A}(0,\mathrm{h}) = \mathrm{F}(\eta', \eta'', \gamma_i)$

$\mathrm{A}(\mathrm{p},\mathrm{h}) = (1 - \mathrm{F}(\eta', \eta'', \gamma_i))^2 \cdot (1 - \mathrm{F}(1/\eta', 1/\eta'', \gamma_i))^{p-1} \cdot \mathrm{T}(\sigma_a, \mathrm{h})$

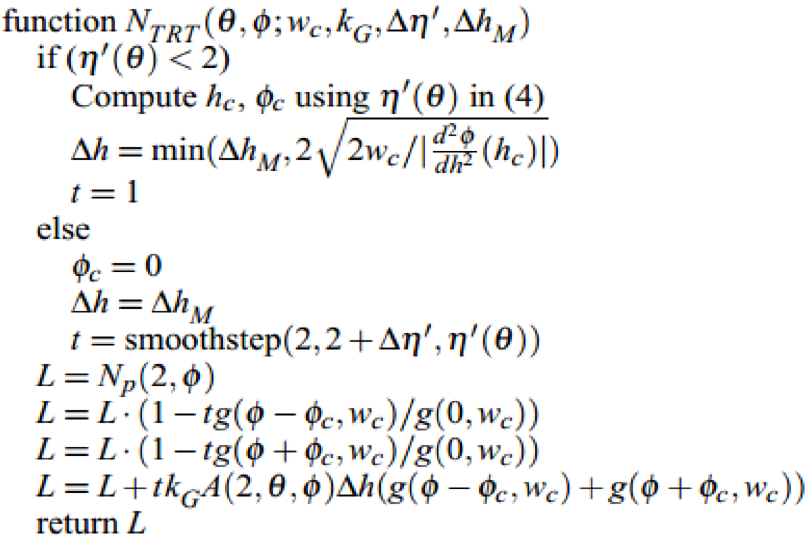
- Finally,

$$\mathrm{N}_p(p,\phi) = \sum A(p,h) \,/\, \left|2\frac{d\phi}{dh}(p,h)\right|$$

■ The approximation of N_{TRT}

- The caustics in the TRT component may produce singularities with infinite intensity.

- We remove the caustic from Ntrt and replace it with a smooth lobe centered at the location of the caustic.



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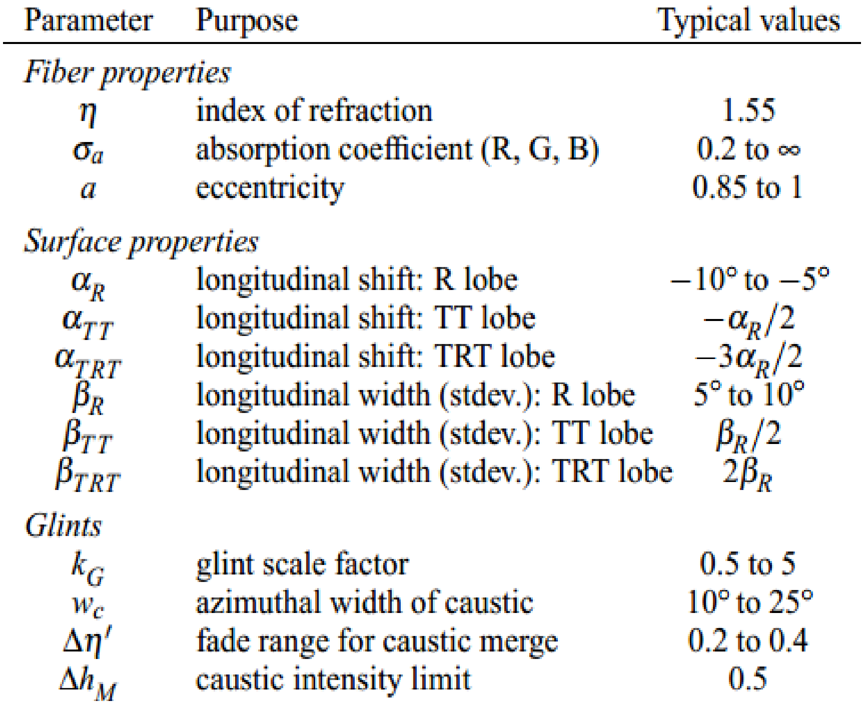
function $N_{TRT}(\theta, \phi; w_c, k_G, \Delta\eta', \Delta h_M)$
  if $(\eta'(\theta) < 2)$
    Compute $h_c$, $\phi_c$ using $\eta'(\theta)$ in (4)
    $\Delta h = \min(\Delta h_M, 2\sqrt{2w_c/|\frac{d^2\phi}{dh^2}(h_c)|})$
    $t = 1$
  else
    $\phi_c = 0$
    $\Delta h = \Delta h_M$
    $t = \mathrm{smoothstep}(2, 2+\Delta\eta', \eta'(\theta))$
  $L = N_p(2, \phi)$
  $L = L \cdot (1 - t g(\phi - \phi_c, w_c)/g(0, w_c))$
  $L = L \cdot (1 - t g(\phi + \phi_c, w_c)/g(0, w_c))$
  $L = L + t k_G A(2, \theta, \phi) \Delta h (g(\phi - \phi_c, w_c) + g(\phi + \phi_c, w_c))$
  return $L$
```

■ The user defined parameters



Parameter	Purpose	Typical values
Fiber properties		
η	index of refraction	1.55
σ_a	absorption coefficient (R, G, B)	0.2 to ∞
a	eccentricity	0.85 to 1
Surface properties		
α_R	longitudinal shift: R lobe	$-10°$ to $-5°$
α_{TT}	longitudinal shift: TT lobe	$-\alpha_R/2$
α_{TRT}	longitudinal shift: TRT lobe	$-3\alpha_R/2$
β_R	longitudinal width (stdev.): R lobe	$5°$ to $10°$
β_{TT}	longitudinal width (stdev.): TT lobe	$\beta_R/2$
β_{TRT}	longitudinal width (stdev.): TRT lobe	$2\beta_R$
Glints		
k_G	glint scale factor	0.5 to 5
w_c	azimuthal width of caustic	$10°$ to $25°$
$\Delta\eta'$	fade range for caustic merge	0.2 to 0.4
Δh_M	caustic intensity limit	0.5